Proof without words: 
Knopp series for $\pi$

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There are many expressions for number $\pi$ as infinite series or infinite product (see for example [1, 2, 3]). In [1] the following series for number $\pi$ is attributed to K. Knopp:

$$\frac{\pi}{4} = \sum_{k=1}^{\infty} \arctan \left( \frac{1}{k^2 + k + 1} \right)$$

Note that in this formula, transcendent number $\pi$ is represented as the infinite sum of transcendent numbers. However a simple visual proof is provided here.
\[ \alpha_k = A_{k+1} - A_k \Rightarrow \tan \alpha_k = \frac{k+1-k}{1+(k+1)k} = \frac{1}{k^2+k+1} \]

References


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