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tmcs@inf.unideb.hu
http://tmcs.math.klte.hu

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Compositions of dilations and isometries in calculator-based dynamic geometry

Jack A. Carter and Beverly J. Ferrucci

Abstract. In an exploratory study pre-service elementary school teachers constructed dilations and isometries for figures drawn and transformed using dynamic geometry on calculators. Observational and self assessments of the constructed images showed that the future teachers developed high levels of confidence in their abilities to construct compositions of the geometric transformations. Scores on follow-up assessment items indicated that the prospective teachers' levels of expertise corresponded to their levels of confidence. Conclusions indicated that dynamic geometry on the calculator was an appropriate technology, but one that required careful planning, to develop these future teachers' expertise with the compositions.

Key words and phrases: dynamic geometry, technology, teacher education.

ZDM Subject Classification: D40, G50.

Solving problems in transformational geometry requires students to manipulate figures, sketch figures with a reasonable degree of accuracy, use lines, directed segments, and points in particular ways, and label sketches effectively to communicate solutions. Many future elementary school teachers have difficulty achieving this level of precision. This difficulty has led to calls to reconsider the ways in which we teach transformational geometry and to teach it in a way that allows students to observe the movements of the figures [9].

Studies support the use of thoughtfully designed sequences of tasks and computer tools to assist in the learning of transformational geometry. In particular, investigative reports of dynamic geometry software in classrooms support the use of activities that engage students in thinking about transformations as components of sequences of motions and as functions that require precise instructions to effect individual motions [2], [8]. Since the mathematics curriculum for prospective elementary school teachers typically includes both isometries and dilations as well as a working knowledge of calculators [1], the study reported here investigated future elementary school teachers' performance on a sequence of geometric transformations that required both dilations and isometries using calculator-based dynamic geometry.

The pre-service teachers who participated in this study were enrolled in a mathematics content course for future elementary school teachers. All prospective teachers in four sections of this mathematics course participated in the study and the same two instructors facilitated the instruction on geometric transformations. Two 110-minute class sessions constituted the treatment involving the dynamic geometry and in each session students participated in both whole class and individual activities involving calculator-based geometric transformations. During the first class session, the instructors use animated displays to introduce translation, reflection, rotation, and dilation. The instructors also explained how to construct images under these transformations. Figure 1 shows an image from the animated presentation for a rotation.

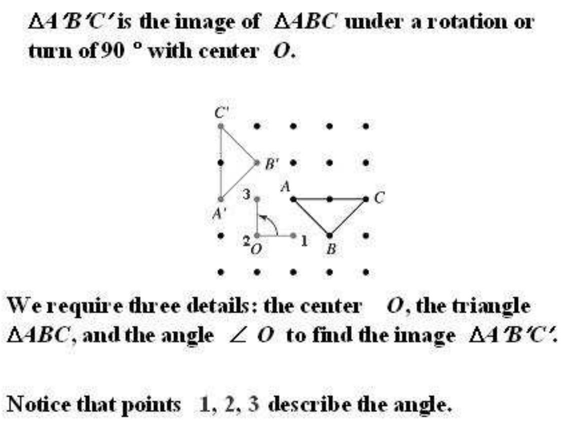

Figure 1. Slide from the animated presentation for a rotation

During the second part of the class sessions, the instructors provided students with opportunities to complete examples of the transformations that corresponded to the animations that were presented earlier. Students were then given a set of instructions that enabled them to view the steps needed for each transformation on the calculators (viz., Figure 2).

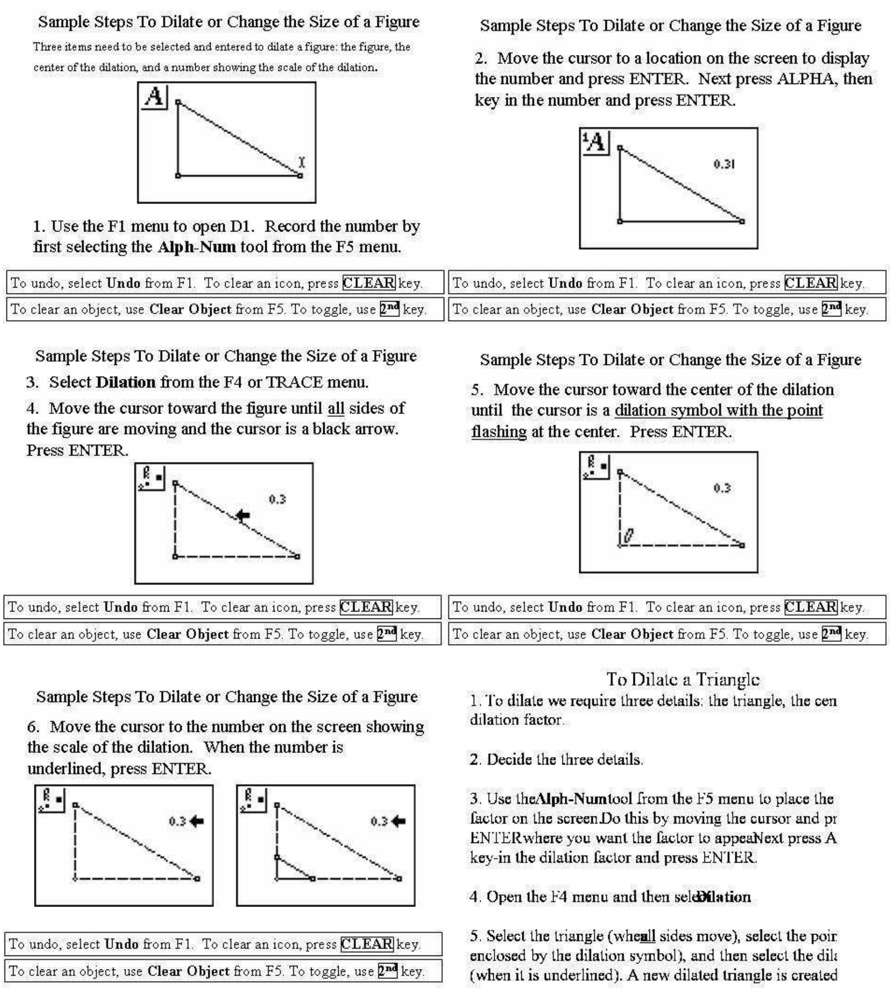

Figure 2. Sample steps to dilate and calculator instruction sheet

Students next individually completed a set of four motion geometry activities. These activities were presented on a handout on which students were asked to open a prepared file in the calculator to begin a particular activity. Instructions for each activity asked the students to perform a transformation in order to transform the pre-image below the line into an image corresponding to the image shown above the line. Students were also asked to self-assess their work by recording a score from 0 to 10 for each activity. Figure 3 shows one of the four activities that were presented on the handout.

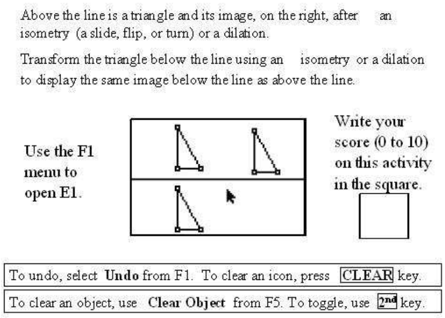

Figure 3. Independent calculator activity for first class session

The second session also began with whole-class instruction and ended with students' individual work on activities that involved transformations on the calculators. During the second session, an instructor reviewed transformations and introduced the composition of transformations by an animated example in which a triangle was rotated through an angle of 180° and then the image was dilated about a vertex using a scale factor of two. Figure 4 shows a slide from this animated example.

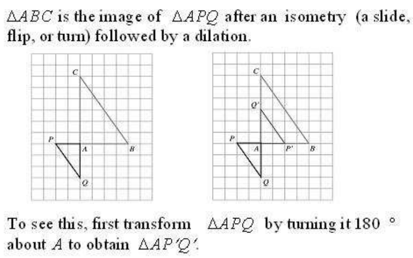

Figure 4. Slide from the animated presentation of an isometry (rotation) followed by a dilation

After the animated example, students practiced a similar composition of transformations using a prepared file on the calculator. For this practice portion of the class, the instructor used both an animated presentation of the steps required for the composition and a projected image of the calculator's display at each of the steps. Figure 5 shows animated slides that instructors used to display the steps for the composition.

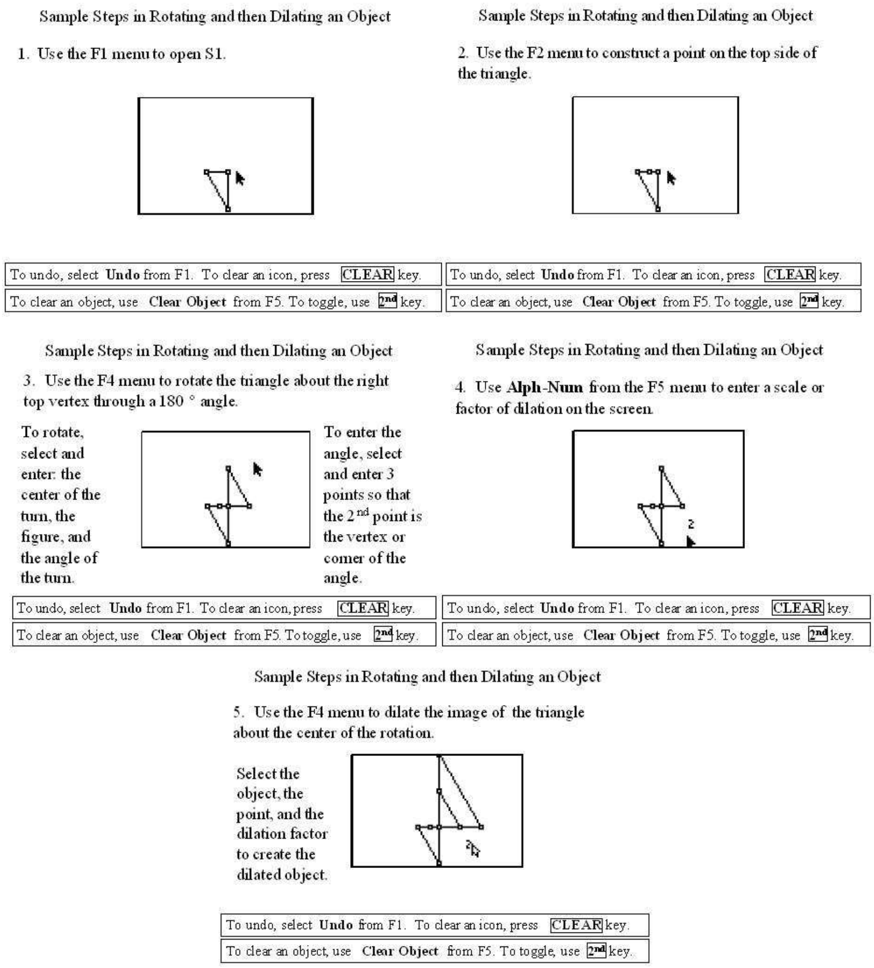

Figure 5. Sample steps to rotate and then dilate

Instructors next distributed a sheet describing eight calculator activities that involved the use of prepared files and compositions of transformations. For each activity students opened a file to display a figure and its image under a composition of transformations. The instructions for each activity asked students to perform an isometry followed by a dilation to construct a corresponding image. Students were again asked to self-assess their work by assigning themselves a score for each activity. Figure 6 shows one of the eight calculator activities that students worked on independently during the second session.

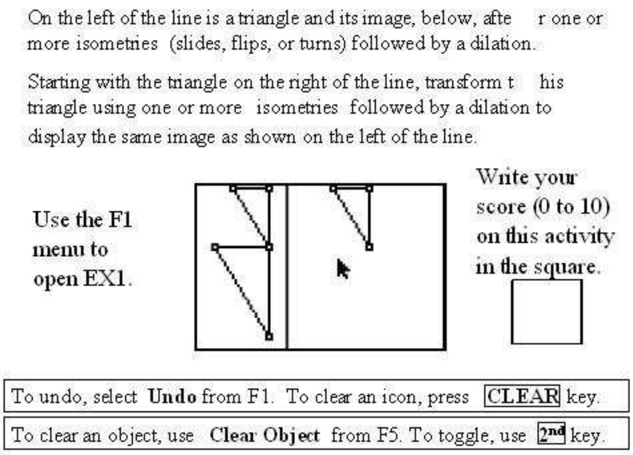

Figure 6. Independent calculator activity from second class session

About a week after the sessions on geometric transformations, students completed a survey about their experiences and answered test items about images or pre-images of transformed geometric figures. Analysis of activity sheets, surveys, and test item results showed that 75 future teachers from four sections of the course completed the activities, surveys and test items. Survey results showed that 95% of students expressed confidence in their abilities to find images or pre-images under geometric transforms. Also, the mean self-assessment score recorded by participating students was 8.74 out of 10.

The future teachers' performance on test items that required finding the image or pre-image of figures under geometric transformations revealed a lower level of expertise than indicated on the self-assessments. Average scores on the test scores on items about rotations, translations, glide reflections, and reflections were 81%, 76%, 69% and 76%, respectively. Figure 7 shows samples of each type of test item with solutions and means.

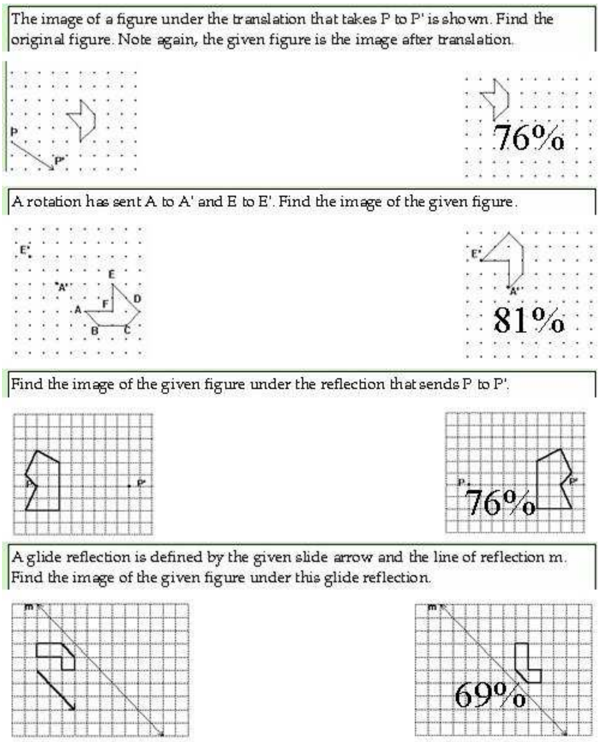

Figure 7. Sample test items (left) and solutions (right) with students' mean scores

Students' performance on assessment items showed modest variation between the four sections of the course and instructors noted two main types of questions during the calculator activities. These questions included how to rotate a figure through an angle of 180° by specifying points on the sides and the vertex of the angle, and how to adjust the image under a translation by moving one of the two points that indicated its distance and direction.

In addition to students' questions concerning how to construct or modify a figure using the calculator-based dynamic geometry, the instructors answered some questions that dealt with determining the angle required for a rotation or the line necessary for a reflection. In answering these questions, it was helpful to represent the transformation using physical objects. For example, rotating the calculator’s case through an angle of 180° provided a convenient and concrete way of illustrating the effect of a half turn in the plane. This was a maneuver that both instructors found useful in explaining the effects of a half turn on an object to these future elementary school teachers during the individual activities.

The instructors concluded that the calculator-based lessons provided an effective and lively way to introduce the prospective teachers to compositions of transformations. Preparing for the lessons required creating and loading the files into student calculators. As a result, the instructors concurred that these sessions required more preparation time. The instructors also concluded that calculators provided a motivational approach that used technology to introduce compositions of transformations. Nevertheless, the calculators had limitations and these were mainly due to the small screen size. Careful planning was necessary to insure that examples and activities could be carried within the physical limits of the screen.

The prospective teachers’ performance on the test items indicated an adequate level of competency and scores on the test items closely matched the perceived confidence levels in working with the transformations. The instructors concurred with the pre-service teachers’ assessment of their abilities with the transformations, and in retrospect, believe the dynamic geometry activities provided an intellectually stimulating use of hand-held technology for these students.

The future teachers’ mean scores on assessment items involving rotations (81%) showed evidence of matching or exceeding their mean scores on items about reflections or translations (76% each), while the mean scores on compositions of motions (69%) showed evidence of being the lowest for the transformations. These results are consistent with findings by Jacobson and Lehrer [7] who reported that practicing teachers’ use of computer tools to learn transformational geometry resulted in small differences in assessed understanding of particular transformations. The instructors also observed only modest differences between classes in terms of competence with particular transformations, another result that matched similar findings by Jacobson and Lehrer [7].

The activities and examples in this study were purposely made to be appropriate for introducing compositions of transformations to these students. As a result, in dilations and rotations, the centers were all situated on the vertices of figures, and in reflections and translations, the parameters (lines of reflection and directed segments) were all vertical or horizontal straight objects. Results reported by Cooper and Shepard [3] and by Edwards and Zazkis [4] indicate that students have more difficulty with rotations through angles that are not multiples of a right angle and with rotations that are not centered at a vertex of a rotated figure. The researchers' experience in the present study also suggests that a greater range of performance measures for the calculator-based activities would result if the straight object parameters for reflections and translations were not only vertical or horizontal.

An extension of the work in the present study would be to consider compositions of isometries and the possible single isometry that would produce the same effect. Due to the small screen size on the calculators that are platforms for dynamic geometry and the lack of a scroll feature on these screens, careful planning would be required to prepare introductory activities for future teachers to investigate compositions of isometries. One viable approach could be to display the pre-image, interim and final images of a composition of isometries on one portion of the screen and ask the students to construct the final image using a single isometry from the pre-image on the other part of the screen. Further investigations could then synthesize the outcomes of these compositions into a table-of-operations format wherein the pre-service teachers record their results and identify patterns that connect compositions of reflections with rotations and translations.

In connecting the role of dynamic geometry to developing understanding of geometric transformations, the present study focused on the future teachers' initial perceptions of transformations as motions performed on objects. A next step in refining these prospective teachers' understanding could be to move them toward thinking about transformations as acting on sets of points in the plane. One way of moving toward this goal could involve using calculator-based dynamic geometry to make concrete the notion that there is one invariant point for rotations, an infinite-number of these points for reflections, and zero invariant points for translations or glide reflections. Students may then gain an understanding that transformations are applied to sets of points in the plane rather than to only an object.

Further insights into prospective elementary teachers' thinking about transformations that enact motions in the plane are apt to require individualized, in-depth interviews. Preliminary work to gain insights into students' understandings of motion geometry has been reported by Flanagon [5] and Hollebrands [6]. These researchers used interview and other data to assess general preconceptions held by mathematics students. Prospective elementary school teachers are a special subset of today's mathematics students, who will use tomorrow's technology to introduce children to motion geometry. By gaining insights about pre-service teachers' understandings of geometric transformations, more appropriate decisions can be made about the best use of instructional technology.

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DR. JACK A. CARTER
DEPARTMENT OF MATHEMATICS AND COMPUTER SCIENCE
CALIFORNIA STATE UNIVERSITY EAST BAY
HAYWARD, CA 94542
USA

E-mail: jack.carter@csueastbay.edu

DR. BEVERLY J. FERRUCCI
MATHEMATICS DEPARTMENT
KEENE STATE COLLEGE
KEENE, NH 03435-2010
USA

E-mail: bferrucc@keene.edu

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