18/4 (2020), 229-240
DOI: 10.5485/TMCS.2020.0507
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Teaching Mathematics and Computer Science

Virtual manipulatives in inquiry-based approach of 3D problems by French 5th graders

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Abstract. The aim of this research is to study the appropriation of a 3D environment by learners in an a-didactical situation of problem solving. We try to evaluate the relevance of the virtual 3D environment in the development of students' cognitive and metacognitive abilities. We implanted a problem-solving activity related to a 3D cube situation with an empty part in the cube in different French primary school areas in May 2019. In the experimental group each learner works individually with a PC-computer where the virtual environment ANIPPO is implemented. In the control group the pupils work in a traditional class environment. We present the results of this pre-experimentation.

Key words and phrases: problem, 3D space, virtual, representation, primary school.

MSC Subject Classification: 97D50, 97U60, 97U70.

Introduction: What contributions from the ANIPPO virtual environment in solving open problems?

In this introduction we specify the context and the objectives of this research. Manipulation in learning mathematics, whether in a real or virtual environment, is an important element in learning mathematics (Moyer et al. 2002, Zilkova et al. 2019). Problem solving continues to occupy a central place in the French curriculum (Ministère 2018). Research in mathematics education in France shows that the theme of problem solving remains topical (Sander 2018, Houdement 2018). Since the work of the French national institute for educational research (INRP), in 1972, with the ERMEL team, problem solving has been at the heart of primary school learning and supports the practice of collaboration between pupils: "the wish is also to develop a cooperative activity among the students and to limit the interventions of the teacher "(Artigue et al. 2007, p.374). A multidisciplinary team bringing together specialists in the educational sciences, cognitive sciences, mathematics education and computer science education have developed this research project.

Theoretical framework: Representations, collaboration and motivation when solving open problems in a virtual environment

Carboneau et al. (2013, p.396) suggests "that simply incorporating manipulatives into mathematics instructions may not be enough to increase students achievement in mathematics". 3D virtual environment could help students as Pittalis & Christou (2010 p.208) indicate: "the backbone of 3D geometry teaching should be tasks that require the mental manipulation of visuospatial relations to conceive and edit geometry properties and take advantage of students' life visuospatial experiences that are produced out-of-school". ANIPPO environment was developed in Second life, a free 3D virtual world where users can create, connect, and chat with others from around the world using voice and text. The problems that are proposed in ANIPPO are formulated and modeled in this 3D virtual environment. Pacurar (2018) shows that the process of identifying the student with his avatar, helps him to build mental representations. Christou et al. (2006, p.169) underline that the interest of using 3D geometry software offers a "semiotic perspective about mathematics as a meaning-making endeavor".

Similarly, the research results show the importance of representations in the teaching and learning of mathematics (Goldin 1998). The ANIPPO environment will allow to work in different representation registers (Duval 2006):

- the register of the ANIPPO virtual world where different treatments are possible: the movement of an avatar in the environment by keyboard commands, the visual control on the screen of a virtual movement of an avatar, visual exploration on the screen of a virtual place (for example by moving the avatar around a virtual solid or inside a virtual solid), the register of natural language written in the windows provided to respond to the posed problems, or in chats between pupils, in which for example pupils can exchange information by referring to references in the virtual space,

- the register of oral language during exchanges between pupils. Duval invites us to distinguish between intra-register processing (such as moving inside the ANIPPO environment, or even counting small cubes by calculating numbers written in figures) and inter-register conversion processing (like the description in natural language on how to move in the ANIPPO environment or the description in natural language of an object located in the ANIPPO virtual space): “Mathematical comprehension begins when coordination of registers starts up. […] Mathematical thinking processes depend on a cognitive synergy of registers of representation” (Duval 2006, p.126). In our research, students use these representations and exchanges in the case of problem-solving activities that occupy a central place in the French curriculum (Artigue et al. 2007). These activities relate to open problems: "Open problems are there defined as problems with a short text, which induces neither a method nor a solution; their solving does not reduce to the direct application of known results or tools; they situate in a context that is familiar enough to the students to make the problem meaningful, and allow them to engage in trials and conjectures ”(Ibidem p.373). Concerning collaboration and cooperation, we will use the theoretical framework of Baudrit's work (2005, 2007). However, in the ANIPPO environment, students will be able to recognize themselves in the virtual space where their avatars circulate and can exchange with each other using an audio headset associated with a microphone, or in writing by using a chat. We will observe if collaborations and mutual aid are set up in this a-didactic environment without professor.

With this theoretical framework we formulate the following two research hypotheses to be verified. H1: The student's immersion in the virtual world reinforces the didactic character by identification with his avatar and by a better relation of the subject to the real world. H2: The reinforcement of the a-didactic situation facilitates the transition to procedural strategies developed from the mathematical knowledge involved.

Methodology: from pre-experimentation to experimentation

The methodology consists in implementing a pre-experiment (in 2019) in 5th grade class (10-11 years old students), the results of which will make it possible to specify an experiment (in 2020). In both cases, problem solving is compared between students in a control group and students in an experimental group. The pupils of the two classes must solve the same problems but under different conditions. Let us describe the pre-experimentation method.

In the control group, the students work in their usual classroom. There are split in different groups (4 to 5 pupils) in which they can collaborate to find a solution. They work with new concrete material (nestable small cubes) and different pictures of a solid, from different points of view (left, front, and above). Although the teacher is present in the classroom, he does not help students to find the solution. A researcher is present in the classroom with the teacher to introduce the general situation, to show how the nestable small cubes are fitting together, and to check that the teacher does no didactical intervention. In the experimental group, subgroups of 4 to 5 pupils work in the computer room. Each student works alone in front of a computer, in which the ANIPPO 3D virtual space environment is installed. There is no class teacher in the room. Students can communicate with each other through a headset with a microphone, or in writing by using a chat available in the ANIPPO environment. Each of the classes (control or experimental) has three work sessions.

A first initial evaluation session has two aims. A first one is to measure changes in the students motivation with a questionnaire at the beginning of the experimentation and another questionnaire at the end of the experimentation. For technical reasons, the motivation test in psychology could not be implemented during the pre-experiment. A second aim is to measure mathematical knowledge and problem solving ability about 3D space. On one hand, it is a question of ensuring that the pupils have the minimum prerequisite knowledge to be able to solve the proposed problems. It is essentially the knowledge of names of usual solids and concepts: vertex, edge and face. On the other hand, a problem to be solved is proposed: it is a question of measuring the performance of each pupil in solving the problem, to possibly observe if there is a link between the pupil's performance in the assessment and his ability to carry out the tasks proposed in the following two sessions. The correction of this initial assessment must be carried out before the other sessions to ensure that the students have indeed had a minimum teaching of the prerequisites.

A second appropriation session aims to discover and take ownership of the work environment. In the control group, it is a session of appropriation of the material constituted by nestable small cubes. Photos illustrate how to fit the cubes. With the little cubes, pupils must build different models proposed by other photos. In the experimental group, this is an appropriation session of the ANIPPO environment. In this session, students do not have to solve any problems. Different screenshots illustrate how to move around the environment, how to change your avatar and how to use the chat.

A third session consists of a problem-solving session. The control group works in normal class configuration, with the material of the nestable small cubes. Each student answers the following two problems on a sheet:

We consider a solid formed by a large cube from which small cubes have been removed. Here (Figure 1) are some views of this solid.

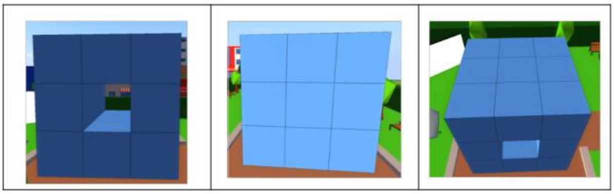

Figure 1.

Three problem levels are proposed with an increasing level of difficulties.

A. How many small blue cubes are needed to form the solid (formed by a large cube from which small cubes have been removed)?

B. Each small cube has square faces. How many square faces does it take to cover the solid (including the inner part where small cubes have been removed)? Instructions: If you wish, you can use small nesting cubes which are provided to you during this class session.

C. The same questions are proposed with a bigger cube (for example 5X5X5 small blue cubes) from which some small cubes have been removed by forming a more complex cavity than in the previous example. The cavity form varies from one group to the other. Different forms of cavities are tested in the groups.

The experimental group works in the computer room, each student having a computer on which the ANIPPO environment is located. The avatar can therefore move in the virtual space (Figure 2), around the large cube as well as inside the large cube.

Figure 2.

Let us summarize the comparison between the two experimental and control groups. In both cases: the situation is a-didactic and the students must write individually the answer to the problems, new material is made available (small nestable cubes versus virtual environment), students can collaborate with each other in small groups of 4 to 5 students (around a table versus in the virtual world, with headphones and microphone), the sequence of events has the same structure within the 3 sessions. From the results of the pre-experimentation, adjustments will be made to the different sessions of evaluation, appropriation and problem solving.

Results of the pre-experiment and impact on the experiment

In the control group of 26 students, 20 answers to the first question A (on the number of small cubes making up the large hollowed out cube) are correct. From the detailed calculations, we can guess two procedures. The calculation "8X3 = 24" suggests that the students cut the large hollow cube into three vertical slices, each slice being made up of a square of 3 cubes out of 3, from which the central cube was removed, that is 9-1 = 8 by slice, and 3x8 in the whole. Another calculation "27-3 = 24" suggests that the pupils counted the number of small cubes in a large 3X3X3 cube, that is 27 cubes from which they removed the 3 cubes from the central part, that is "27-3 = 24". The second question B, for which the correct answer is 64, is answered correctly by only 4 students. This question does not seem to have been understood. For example an incorrect answer is 27X6 = 162, because the previous 27 cubes are each covered by 6 faces. In the experimental group, for the two questions, A and B, no correct answer is given. None of the groups succeeded in answering to question C. We are first very surprised about the difference in success rate between the control group and the experimental group.

By interpreting these results, here are the proposals for improvements to the conditions of the experiment, first of all concerning the two groups. There are not the same pupils and not the same teachers for both groups. For the change of teachers we could split the class in two groups although it is more complicated to organize because we have not enough computers for a half class. For the change of pupils, in the experimentation, a bigger number of pupils will reduce the natural fluctuation. Another traditional difficulty is related to understanding the problem as well as memorizing the problem statement. In the control group the pupils have a permanent paper sheet where the problem questions are written. For the next experimentation we decide that the experimental group could access permanently to the written question by clicking with the mouse on the screen. To reduce the risk of misunderstanding the questions we will propose for both groups at the end of the second appropriation session the following question to check if the question is well understood and to identify the pupils who already have difficulties in a simpler question:

"Here (Figure 3) are different views of a solid.

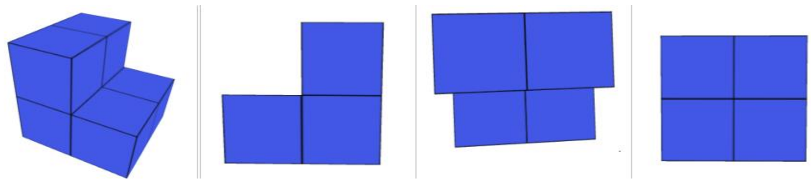

Figure 3.

A) How many small blue cubes do you need to form the solid? John answers: you need 5 small blue cubes. Do you agree? Explain why?

B) How many small squares does it take to cover the solid? John answers: we need 11 small blue squares to cover the solid. Do you agree? Explain why?"

None of the experimental or control groups answered the question C. We interpret that because of a lack of time and the difficulty of the question. But some pupils of the experimental group entered the cavity to explore some counting procedures. It will be interesting to observe if with more time they can success in answering the question. For experimentation we will separate in the experiment the third session into two sessions: the first concerning the hollowed cube 3X3X3 and the second the hollowed cube 5X5X5.

Let us explain our interpretation of the difficulties linked to the ANIPPO virtual environment. On one hand some pupils have difficulties appropriating the ANIPPO environment during the 1.5 hour appropriation session. Recordings of the movements of avatar screens show that many students have difficulties moving around the environment and have difficulties performing some actions. Likewise, the pupils seem not to take the time to read the instructions posted in the environment. Would it be better if an audio message would be delivered to the passage in front of a detector?

On the other hand the situation in the ANIPPO environment appears very playful, to the point that many students seem to forget the initial problem-solving objectives: some students have fun exploring the environment and playing hide and seek between avatars as evidenced by audio exchanges between students. To help the pupils to persist in the problem-solving task, the virtual environment will be simplified to make it less distractive and regular reminders on the problem-solving task will be broadcast (audio and video) in the environment.

Regarding hypothesis H1, the student's immersion in the virtual world seems to reinforce very strongly the a-didactic character of the situation, by identification with his avatar and by a better relation of the subject to the real world. This immersion even appears as an obstacle in comparison to the control group. There should therefore be a moderation of the didactic character: perhaps the identification with the avatar should be more distant to avoid distracting the student too much. Should regular reminders be given of the problem-solving objectives? Should we allow a longer appropriation time so that the problem solving session is no longer disrupted by an incomplete appropriation of the environment? Finally, as far as the control group is concerned, does a phenomenon of copying between pupils exist which would be favored by the whole class environment and by devices of the answer sheet type to be completed, which are not without recalling the evaluation systems? Perhaps it would be interesting to note the geographical arrangement of the pupils to observe if a phenomenon of similar responses (correct s or not) exists for neighboring pupils?

Regarding hypothesis H2, for the few students who gave incorrect answers to the first question, it was difficult to observe their procedures. During movements in the virtual environment, information is exchanged in the written register, in particular the display of the problem statement and the student's production of a solution to the problem. Is this transition from the action register to the written register problematic in a virtual environment? Is the attention and motivation sufficient to memorize the written statement while traveling and to set up explorations of the virtual environment with the aim of solving the problem? It might be interesting, once the student has proposed an answer, to record the oral justifications for his result, either by a process of recording part of the ANIPPO environment, or by an external interrogation. A system of better traces of the justifications for the responses could also be proposed for the control group. The ANIPPO virtual environment appears complex and not very intuitive in its design and use: would simplification allow faster and better appropriation?

Many technical problems appeared in the two places (Lille and Marseille) where the pre-experimentation occurred: these problems will have to be resolved before the experimentation. But it is a characteristic of lessons involving ICT that technology dominates didactic action (Trestini et al. 2006). Concerning the second problem, very few students succeeded in the control group and none was successful in the experimental group, therefore the question of the level of difficulty of the geometry problems in space is raised. However, the work of the ERMEL team (Douaire et al. 2009) seems to show that this type of problems is accessible in primary school. So is this a problem of understanding the statement? Have the students in the control group and the experimental group done enough work in this area? Possible problems should be foreseen where the difficulty appears more gradually. The evaluation planned for the first session could not take place and therefore did not provide additional lighting. Perhaps an additional problem-solving session should be planned to suggest a gradual increase in the level of difficulty of the problems.

Conclusion and perspective

Up to the pre-experimentation stage, we have described a research device for solving open problems in primary school in the 3D ANIPPO virtual environment. We have shown the importance of several factors: the technical context which should guarantee the smooth running of the experiments, the time for getting used the environments (concrete or virtual) which must be sufficient, in particular for the rich and complex ANIPPO environment, the registers of representation used for the exchange of information, the observation of justifications for students' strategies and procedures. In particular for the understanding of the problem statements and for the observation of the strategies put in place to solve them and of the procedures leading to the solutions, additional devices must be proposed: for example observers during pupil work, recording of screen work or pupil interview after problem solving session. Under the conditions of the pre-experiment, the ANIPPO environment appeared unfavorable for the resolution of open problems in a 3D space. Would a modification of the conditions of experimentation and the scenarios used in this environment make it possible to favor the resolution of open problems and to see the appearance of strategies and procedures of resolution specific to this environment?

Karsenty and Bugmann (2018) argue that an unsupervised practice of a 3D game does not guarantee a benefit for learning. It is necessary in our experimentation to come to define a framed and structured practice which allows to switch from an initial motivation for the 3D game to a positive factor to solve problems. Future experimentation will develop pre and post questionnaires to analyze the students' motivation about playing and learning based on research of Fenouillet, member of ANIPPO research team (Lieury, Fenouillet 2013). "The game is then no longer a means of achieving or evaluating an educational objective, as in the case of a serious game, but rather the objective which makes it possible to mobilize, in the learner, the skills and resources necessary for the design of a game. This is a transposition of Dewey's learning by doing" (Plante 2016, p.74, translation from French by authors).

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