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A retrospective look at discovery learning using the Pósa Method in three Hungarian secondary mathematics classrooms

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Abstract. While the Pósa Method was originally created for mathematical talent management through extracurricular activities, three "average" public secondary school classrooms in Hungary have taken part in a four-year experiment to implement the Pósa Method, which is based on guided discovery learning of mathematics. In this paper, we examine the students' and teachers' reflections on the Pósa Method, and how student perspectives have changed between their first and last year of high school. Overall, teachers and students had a positive experience with the Pósa Method. Furthermore, our research indicated that this implementation has met several objectives of the Pósa Method, including enjoyment of mathematics and autonomous thinking.

Key words and phrases: Pósa Method, guided discovery, discovery learning.

MSC Subject Classification: 97D40.

Introduction

The Pósa Method is a pedagogical approach that is based on discovery learning. It is mainly used to help advanced mathematics students in Hungary cultivate their mathematical skills. Developed by prominent mathematics educator Lajos Pósa since the late 1980s, its fundamental objective is to make students happy while thinking about mathematics. It also aims to cultivate a student's ability to think independently and deeply about mathematics (Győri & Juhász, 2017). To achieve this end, the Pósa Method presents students with problems in a way that builds upon their previous experiences and encourages them to notice connections between seemingly different areas of mathematics (Katona & Szűcs, 2017).

As the Pósa Method has mainly been used in extracurricular programs for talented students, there is an interest in whether the benefits of the Pósa Method can be experienced by more students with varying levels of mathematical capability in different contexts. As such, in 2017, the Content Pedagogy Research Program of the Hungarian Academy of Sciences supported a four-year-long experiment, conducted by the MTA-Rényi Research Group on Discovery Learning in Mathematics, which supports the implementation of the Pósa Method in more typical secondary school environments. There has been very little research about the efficacy of the Pósa Method in a more typical classroom, so the experiment offers a unique opportunity to investigate the role the Pósa Method can serve in these populations. Hungarian students typically attend high school for four years with the same teacher. As the experiment ends in 2021, and students approach the end of high school, this paper takes a retrospective look at three teachers' implementation of the Pósa Method and their students' opinions of discovery learning. Our study employs mixed methods techniques. Namely, the qualitative portion consists of teacher interviews and open-ended survey questions. We also apply statistical procedures to determine how students' attitudes, perceptions, and beliefs evolved over the four years.

The paper begins with a literature review containing some features of discovery learning and the Hungarian education system. Next, we discuss our motivation, research questions, and methodology used for data collection. Finally, we present our findings and conclude with limitations of our research and directions for future work.

Literature review

Guided discovery

The Pósa Method has been compared to several discovery based approaches all over the world (Artigue et al., 2020) such as sharing characteristics and values with inquiry based learning: "the central role given to the formulation and solving of significant problems and questions, the autonomy given to students in the inquiry process, [and] the importance attached to the quality of their mathematical activity" (pp. 78–79). As Gosztonyi (2020) describes, "the conception, named teaching mathematics by Guided Discovery" (p. 12) in the Hungarian context refers to the Complex Mathematics Education reform of Tamás Varga; however, the term is coherent with existing international classifications as well.

According to Honomichl and Chen (2012), learning by discovery is a pedagogical strategy which helps the learner "create and organize knowledge" (p. 615). In particular, the facilitation of problem solving experiences encourages the learners to improve their understanding and deduce strategies by building upon their previous knowledge and experiences. Furthermore, empirical findings suggest that guidance should be included in discovery learning, which yields guided discovery, a pedagogical framework which Honomichl and Chen (2012) describe as "similar to Vytgosky's idea of scaffolding: guidance that is dynamic and responsive to to the learner's current state of experience and ability, with inexperienced learners receiving greater guidance or supervision and experienced learners receiving less intervention" (pp. 615–616).

Honomichl and Chen (2012) write that the objectives of guided discovery can be supported by "(1) strategic presentation of materials (2) consequential feedback, and (3) probing questions and self-explanations" (p. 616). These strategies not only give an opportunity to incorporate guidance into discovery learning, but together they also let the guidance proliferate across various time slices of the learning process. In particular, (1) "focuses on initial problem/task experience," (2) "reflects immediate post-solution assessment," and (3) "deals with subsequent refinement and restructuring of concepts and strategies" (p. 616). As will be explicated later, the Pósa Method exemplifies these qualities.

History of Hungarian mathematics education and the emergence of the Pósa Method

Stockton (2010) notes that the work of individual mathematics educators can make an impact for generations in Hungary. Some of the most notable influences prior to the "New Math" movement were made by Loránd Eötvös, whose influence led to the creation of the Society of Mathematics and Physics and fostered a culture that focuses on problem solving. Lipót Fejér, the first in Hungary to have a mathematical school organized around their own person, and the Circle of Karácsony, a group of mathematicians, educators, and thinkers who discussed education in the 1940s and played a role in future reforms (Stockton 2010; Gosztonyi, 2016). Fejér's pedagogical approach emphasized that the student should think that "even he or she could have done that," which was repeated among the Circle of Karácsony (Gosztonyi, p. 75, 2016). They maintain that education should guide the learning process with intuition, dialogue and experience. Furthermore, they believe formal language should be limited, and art, play, and creativity should be prioritized (Gosztonyi, 2016).

The latter half of the 20th century saw national and global changes in mathematics education due to the "new math" movement (Kilpatrick, 2012). Gosztonyi (2015) notes that Hungarian reform focused on the relationships between different areas of mathematics, the generalization and abstraction of mathematical ideas, and heuristic methods. One of the leaders of "complex mathematics education" reform was Varga, a member of the Karácsony circle (Gosztonyi, 2016). At around the same time, Pósa attended the inaugural special math class at Fazekas Gimnázium in 1962, an option which had only recently become politically viable (Győri & Juhász, 2017, pp. 4–5; Stockton, 2010). Later becoming a mathematician, Pósa was inspired by Varga and Surányi, members of the Karácsony circle, and became interested in mathematics education. In particular, Pósa admired their "complex teaching" which focused on independent work, cultivation of problem solving skills and discovery as a vehicle for teaching mathematics (Győri & Juhász, 2017).

According to Katona and Szűcs (2017), "the main goal of the Pósa Method [...] is to offer the opportunity for students to be happy by thinking on interesting mathematical problems and discovering mathematical connections." (Katona & Szűcs, 2017, p. 18). Moreover, Győri and Juhász (2017) note the Pósa Method largely focuses on cultivating independence and discovery. To achieve these results, the teacher must understand children and their knowledge in order to develop problems and conditions for good thinking. When a teacher implements the Pósa Method, students must be given enough time to think, allowing student learning to resemble the work of research mathematicians. While group work is important, a core objective of the Pósa Method is to help students find solutions through their own efforts. Consequently, Pósa implemented special rules which prevent solutions from being spoiled, and students can be placed in different rooms to make sure that fast solvers do not discourage or intimidate others (Győri & Juhász, 2017). As students work through problems, the Pósa Method approach encourages students to ask both conceptual and follow-up questions in a way which can help them understand what makes their material interesting, learn the value of a good question, have the prerogative to make mistakes without any embarrassment, and engage in student-teacher dialogues (Győri & Juhász, 2017; Katona & Szűcs, 2017).

To facilitate these components, instructors using the Pósa Method create a web of problem threads which guide students towards new mathematical ideas and connections between different topics. A "thread" is defined as problems which are related to one another (Katona & Szűcs, 2017). Katona and Szűcs (2017) write "One aspect of this connection is that the problems may be, to a different extent, built on each other, meaning that the anticipated or regular solution to one problem (problem A) needs certain ideas, methods, types of mental representations, etc., or in general, any separable element of thinking, that can be made more easily available for students by thinking on and solving another problem (problem B), or even some other problems" (Katona & Szűcs, 2017, p. 20). The idea which links two problems in a thread is called a "kernel". Students work through multiple threads simultaneously, and sometimes these threads intersect. Together, the threads make a web of problems (Katona & Szűcs, 2017).

History of the Pósa Method's implementation in the Hungarian educational system

According to Győri and Juhász (2017), Pósa first created a camp where "15-20 clever seventh-graders [...] could work on mathematical problems together in a concentrated way" in 1988 (Győri & Juhász, 2017, p. 197). Since 2008, a robust structure has emerged, and the Pósa camps have expanded significantly to include several parallel groups of students, weekend camps, and summer camps. While students are accepted into summer camps on the basis of their competition results, there are multiple avenues, including a teacher's recommendation, attendance at a school that Pósa might visit, or a parent's request, to be screened by Pósa for admission into a weekend camp. All children must also show their commitment to mathematical work by solving a set of questions (Győri & Juhász, 2017). In addition to being taught with the Pósa Method, students participate in team contests and meet "evening guests" who help "broaden the way of thinking" (Győri & Juhász, 2017, p. 101). These camps are run by Pósa, his disciples, and helpers who were once participants, and so the essence of his pedagogy is well maintained (Győri & Juhász, 2017).

In 2017 the MTA-Rényi Research Group, funded by the Content Pedagogy Research Program of the Hungarian Academy of Sciences, began a four-year implementation of the Pósa Method in three public secondary school mathematics classes (Matzal et. al, 2020). In public high schools in Hungary, teaching is guided by the Hungarian National Core Curriculum regulating content in two years' cycles, and the requirements are further specified at the local and school level.

Teachers are responsible for charting a path towards completing the requirements given. Since teachers in Hungary stay with one group of students for the entire four years, they have more flexibility with regards to how and when the required topics are covered (Stockton, 2010). The three experimental classes are required to cover the content set by the Hungarian National Core Curriculum; however, they are exempted from following the schools' local curricula. A qualitative research study by Matzal et. al (2020), which also investigated the experimental classes from the MTA-Rényi Research Group, noted that teachers adjust the gaps in difficulty between problems on a thread according to the level of their students, for example by adding or leaving out problems. There are one to five problems to solve during a typical 45-minute class period. However, there are also 90-minute-long "double classes" which were observed in the study. During the observed classes, students were given 45 to 50 minutes to work on about five open problems individually or in groups, which was followed by discussion, and about one to three problems were assigned for homework at the end of class. It is possible that there is no time allocated to discussion if students have not solved the problems. While students knew that they were learning with the Pósa Method, they were not explicitly told what were the problem threads.

Methods

Research motivation

Thus far, the Pósa Method has been implemented for gifted mathematics students in Hungary. While Pósa believed that the joy of discovering mathematics should belong to all students, he also expressed concern that more average mathematics students might be discouraged due to the time needed to think about problems (Győri & Juhász, 2017). Moreover, given the Hungarian cultural tradition and the complex math education reforms which laid the foundation for the Pósa Method, there are also questions and doubts about where else and how the Pósa Method can be applied (Stockton, 2010).

Research questions

This research investigates how students with more average mathematical performance respond to the Pósa Method. The current study contributes to this body of literature by investigating the perceptions, beliefs, and attitudes of students who have learned with the Pósa Method for four years. The study was guided by the following research questions: (1) What are student perceptions about the characteristics of Pósa Method after four years? (2) What are changes in student perceptions about mathematics and the Pósa Method after four years?

Setting and participants

Our team investigated three grade 12 classrooms which participated in the four-year experiment. These classrooms will be referred to as Classrooms 1, 2, 3 with Instructors 1, 2, 3, respectively. Classroom 1 is in a bilingual vocational school with a chemistry pathway, and Classrooms 2 and 3 are located at the same general secondary school, which is considered one of the top ten high schools in Hungary. This school hosts special math classes, too, but Classrooms 2 and 3 are not specialized in math. In grade 9 and 10 these classrooms had three math lessons a week, but in grade 10 the vast majority of the class decided to choose to take the advanced level high school leaving Matura Exam, thus they have five lessons a week in grade 11 and 12. Due to the fact that students in Classroom 1 study math in English, they have four math lessons through all the four years of high school. Classroom 1 has 16 students, Classroom 2 has 14 students, and Classroom 3 has 16 students.

The experiment was led by one of Pósa's former students, who recruited two colleagues, a fellow Pósa camp instructor and his former university teacher trainee, both trained in implementing the Pósa Method. As part of the experiment, these three instructors collaborated to design a curriculum which utilizes the Pósa Method. Instructors modify appropriate material based on student needs and abilities.

Due to the COVID-19 pandemic, Hungarian secondary schools have been online from March 2020 to May 2021, excluding a brief return to in-person learning from September 2020 to November 2020. The instructors adapted their teaching methods to accommodate online learning, and our team modified our research instruments accordingly, as elaborated in the next section.

Research instruments

A mixed methods research design with a concurrent triangulation approach was used to address the research questions. In a concurrent triangulation approach, "the researcher collects both quantitative and qualitative data concurrently and then compares the two databases to determine if there is convergence, differences, or some combination" (Creswell, 2009, Chapter 10). In our case, qualitative data (interviews and open-ended survey questions) were collected to support the findings of the quantitative component (survey). A description of the research instruments is given below. Our team interviewed each of the instructors in the respective classrooms. Each interview was conducted over Zoom and lasted approximately one hour. Interview questions discussed include the delivery of the Pósa Method, teachers' expectations, reflections about the experiment, and the Matura Exam (the results of which will not be discussed in this paper). Our interview with Instructor 1, who also served as a member of the research team, was done as a pilot interview. Appropriate changes were made to the interview protocol, including modifying ambiguous statements and reordering questions. Interviews of Instructors 1 and 2 were given in English while the interview from Instructor 3 was conducted in Hungarian with a translator and subsequently transcribed to English. While our research questions sought to investigate students' views of the Pósa Method, the teacher interviews increased the reliability of our data. Since instructors construct course material, assess student work, and interact with students on a daily basis, they have great insights into student strengths, weaknesses, and evolution throughout the four-year experiment. As a result, the interviews helped corroborate our findings on the student survey.

The student survey was created using Google forms and distributed to students through their instructors. Our survey was modeled after Kile et al. (2018), as they surveyed the same group of students, and we wanted to be able to compare their answers from 2018 to 2021. Their survey consists of six constructs: background, discovery learning expectations, reasons for participation, workload, interests/math preferences, and grades. The constructs in our survey include: background, Pósa Method, interests/math preferences, Matura Exam, and remote learning. Since our research questions focus on student perception of the Pósa Method and mathematics in general, and how it has evolved over the four year experiment, our math background and math preferences sections are identical to Kile et al. (2018). Each of the questions in the math preferences section used a five-point Likert scale from "I completely disagree" (1) to "I completely agree" (5). In addition, questions from our Pósa Method sections were inspired by Kile et al.'s (2018) discovery learning expectations, workload, and grades constructs. Our study sought to investigate students' perceptions of the Pósa Method. However, through teacher interviews, we found that every class had experienced remote learning for the majority of the 2020–2021 academic year. Given the emphasis of dialogue when implementing the Pósa Method, it was clear that the COVID-19 pandemic disrupted its implementation. Since our study sought to highlight the Pósa Method, the wording of questions tried to delineate between the first couple of years (in person) and remote learning. In addition, a "Remote Learning" construct was created to determine how disruptive the online format was for students.

Data analysis

Our survey required the use of both quantitative and qualitative analysis. For questions involving quantifiable data (Likert-scales, percentages, time spans, yes-no questions, etc.) descriptive statistics were used, specifically mean and standard deviation. If a question was posed in Kile et al. (2018), we also conducted an independent sample t-test to investigate whether the additional years of experience had changed student views.

Open-ended survey questions were analyzed using the constant comparative method (Strauss & Corbin, 1991). Specifically, each response was given an initial description. New responses were either placed in the already existing category, used to create new categories, or help modify an existing category. If a sufficient number of responses were given, we also noted the frequency of each description.

Results and discussion

Before addressing the results and discussions of our research questions, it is important to note that online learning had a sizable impact on the Hungarian school system. As mentioned in the literature review, Hungarian schools have been mostly online since March 2020, and teachers have been tasked with the demanding responsibility of adapting their classes to this setting. Although 12 of the 44 students reported that remote learning had little to no effect on their math class, many students reported that it was harder to pay attention and that they felt less motivated and engaged. As mentioned in the methods, due to the disruptions and limitations of online learning, the survey asked students to reflect upon their first three years of high school.

Since the study covers several topics, the findings will be presented in several sections. First, teacher and student perspectives on discovery learning, specifically the Pósa Method, are shared. Second, results from survey questions about student attitudes towards mathematics during their last year of high school are analyzed.

Lastly, the results of some questions are compared with results from Kile et al. (2018) of 47 students, 44 of whom participated in this study (one student left Classroom 1 and thus the experiment in grade 10, and one student each from Classrooms 2 and 3 did not fill out the survey in 2021).

Thoughts on discovery learning

During the teacher interviews, the teachers shared similar reasons for supporting the Pósa Method. All three mentioned that one of their favorite aspects is the joy students experience when they discover an answer. Additionally, they reported that the Pósa Method encourages providing ample time for mathematical exploration rather than rushing through the material. Teachers felt that most students had a positive experience learning with the Pósa Method, and this was generally reflected in student responses to the survey.

In order to gauge student interest in discovery learning, the survey asked students to rate learning mathematics with discovery learning on a scale of 1 (horrible) to 5 (great). There were 40 students (91%) that answered "5 - great" or "4 - good" (Appendix A). When asked why, one student stated "because it's not that I'm quickly taught the material, but I have the time for understanding it for myself. And the feeling after your own success, you cannot get from anywhere else". Another student answered, "It's much more exciting than learning in the traditional way." Thus, it is plausible that the implementation of the Pósa Method met one of its primary objectives, which is to make students happy while they think about mathematics.

Students were also asked to rate the difficulty of discovery learning. No student chose 1 (very easy) or 5 (very difficult), but most students felt that discovery learning was an average level of difficulty (mean = 2.89, standard deviation = .69) (Appendix A). When asked why, one student shared that they thought discovery learning was moderately easy "because it doesn't teach you formulas, but how to think, and that's what you need to solve the problems".

During the interviews, the teachers shared that the main goal of the Pósa Method is student enjoyment of learning mathematics. Consequently, teachers prioritize creating a positive and comfortable classroom environment. When students were asked for their favorite aspects of their math classes, a common response (7 instances) was the atmosphere, which speaks to the overall enjoyment and comfortable environment. Many students commended the friendly and informal atmosphere, and one student shared "the atmosphere was motivating to solve the problems, I could progress at my own pace, it is a really good feeling when you figure out how to solve a problem". Another focus of the Pósa Method is independent thinking and intellectual freedom. That is, students are encouraged to choose which problems to work on, make mistakes, and think autonomously (Győri & Juhász, 2017). Nine students mentioned freedom or independence as their favorite aspect of class. One student stated "the class didn't go according to a fixed schedule, but everybody could work on the problems they wanted to and in the pace that was suitable for them." Although a special emphasis is placed on students finding the answers themselves, another important facet of the Pósa Method is that students discuss problems in groups (Győri & Juhász, 2017). Accordingly, it was no surprise that students reported that one of their favorite aspects of the class was group work (10 instances). One student responded "[my favorite aspect was] working in pairs as it gave me new ideas and I got to know different ways of thinking".

Despite its popularity, group work was also listed as some students' least favorite aspect (3 instances). A couple of reasons provided were that they had different levels of preparation and that some of their classmates were difficult to work with. In addition, students mentioned that there was a lot of noise and distraction (7 instances). Students reported that "classmates often talked in class in a way that disrupted the process of solving problems" and "those who did not pay attention and therefore did not understand the tasks held up the others." This speaks to the double-edged nature of group work, as it can lead to distraction and interdependence, which is not conducive to all learners' needs. The most common criticism was that students were tired or got tired over the course of a 90-minute lesson (8 instances). It is important to acknowledge that noise, tiredness, and the 90-minute lesson could be attributed to faults in the school or classroom structure and schedule rather than the actual implementation of the Pósa Method. In addition, a few students reported that their least favorite aspect of class was having short tests (4 instances). Note that different teachers had slightly different assessment structures, and only Instructors 2 and 3 (28/44 students) gave their students short tests for grades.

In addition to students' least favorite aspects, the survey also asked students to share suggestions for improvement. Responses to these questions shed light on a few main concerns. Three students pointed out that overarching goals were not presented in the beginning of the class and students were not told the lesson plan ahead of time. Five students reported struggling to find the starting idea necessary to complete a problem. Six students across all three classes felt there was not enough repetition or practice of concepts and would have preferred more tasks on each topic. For example, one student wrote "sometimes you have to keep many things in mind at once and without a lot of practice it is sometimes difficult". Interestingly, all of these concerns relate to the problem thread structure described in the literature review, which is a very central and intentional part of the Pósa Method (Katona & Szűcs, 2017; Matzal et al., 2020).

Shift in math preferences

As mentioned in the methods section, students were presented identical survey questions from Kile et al. (2018) in order to gauge how students' perspectives of mathematics have shifted between their first and last year of high school. Students were asked about how interested they were in their math class, the extent their grades reflected their effort and math knowledge, and several questions about their math preferences.

The following questions did not have a statistically significant change at the .05 level between 2018 and 2021, as determined by an independent sample t-test (Appendix B). Students rated how interesting they found their math classes on a scale of 1 (very uninteresting) to 5 (very interesting) and found them rather interesting; there was a 4.21 average in 2018 and 4.02 in 2021. On average, students felt that their grades reflected their effort and their math knowledge, though they felt they should have earned a slightly lower grade based on their effort and knowledge in 2021 compared to 2018. Students were also asked to rate several statements using a five-point Likert scale. "I understand the math I learned from 9th to 11th grade" and "I like coming up with and solving my own math questions" each had a small increase in agreement, with a mean increase of .17 and .18, respectively. On the other hand, "I am good at solving difficult math problems" had a slight decrease in agreement, with a 3.47 average in 2018 and 3.27 in 2021.

There were nine statements that showed a statistically significant difference between 2018 and 2021, particularly with respect to aspects of the Pósa Method and discovery learning (Appendix C). While there were some significant changes in attitude which align with the objectives of the Pósa Method, there are others which conflict. This section discusses these changes.

Students in the 2021 survey agree much more with the statements "In life, a problem can be approached from different angles" (M=4.8, SD=0.59) and "If I face a problem, I try to understand it from different angles" ($M = 4.5, SD = 0.82$), then they did in 2018 ($M = 4.43, SD = 0.89$), $t(89) = -2.4, p = .02$ and

$M = 3.3, SD = 0.88, t(89) = -6.7, p < .001$, respectively). The increase in agreement for both of these questions indicates not only a nearly unanimous agreement with this already generally acknowledged statement, but also a proactive decision to incorporate this knowledge into their approach. In particular, the notable increase in agreement with the second question "If I face a problem, I try to understand it from different angles" substantially closes the gap in agreement between how much students believe that problems can be approached from different angles and how much they try to understand a problem from different angles. As implementation of the Pósa Method encourages students to come up with their own solutions to problems that might require experimentation from multiple perspectives and share them with the class, it is plausible that students are further inclined to approach problems from different angles. While it is not necessarily true that students will try to approach problems from multiple angles after they discover that there are almost always multiple solutions, the ethos of the discovery-based, experimental, and collaborative structure of the Pósa Method is likely to create an environment where exploration across ideas is incentivized and more likely to yield a solution.

Similarly, in 2021 students agree much more strongly with the pair of statements "It is possible to arrive at the correct solution in multiple ways" ($M = 4.77, SD = 0.6$) and "I can arrive at the correct solution in multiple ways" ($M = 3.91, SD = 0.91$) than in 2018 ($M = 2.6, SD = 1.26$), $t(89) = -10.6, p < .001$ and $M = 3.32, SD = 0.98, t(89) = -2.97, p = .004$, respectively). It reflects not only the knowledge of multiple solutions, but also the corresponding increase in confidence which ideally comes with such mathematical insight. While the increase in the students' confidence to arrive at the correct solution in multiple ways is not as substantial as their belief in the existence of multiple ways, it must be noted that students were faced with more open ended and tricky questions as time progressed. The realization of the breadth of problems which are beyond their reach, which may be beneficial if approached with a healthy attitude, may have led to the consequent evaluation of their personal capacities.

Students agreed significantly less with the statement "I like practicing the same procedure through multiple math problems" in 2021 ($M = 3.86, SD = 1.03$) compared to 2018 ($M = 4.51, SD = 0.75$), $t(89) = 3.4, p < .001$. This suggests that students might prefer the dynamic quality of the problem threads over practicing the same procedure for several days. Students in the 2021 survey also more strongly agree with the statement "I enjoy figuring out solutions by myself" ($M = 4.43, SD = 0.85$), than they did in 2018 ($M = 3.28, SD = 0.97$), $t(89) =$

$-6.1, p < .001$. This gives evidence for the idea that students develop positive sentiment towards independence as they practice the Pósa Method. It is thus plausible that cultivating independence is interwoven with one of the primary objectives of the Pósa Method, making students happy while they think about mathematics.

Students rated how interesting they found the math problems on a scale of 1 (very uninteresting) to 5 (very interesting). Although the Pósa Method aims to provide students with interesting problems which grip their attention, students showed a significant decrease in their agreement with the statement "How interesting were the math problems?" in 2021 ($M = 3.89, SD = 0.72$) compared to 2018 ($M = 4.3, SD = 0.83$), $t(89) = 2.5, p = .013$. We offer two hypotheses for this decrease. The first hypothesis is that a roughly monotone level of "interesting" will eventually be perceived as the norm. As a result, problems that were once "interesting" become standard. The second hypothesis is that the sheer difference between how interesting the problems were in their last year of middle school and first year of high school led to strong affective responses which are impossible to replicate for a span of four years. Given that students still find questions interesting on the whole despite these speculations, the decrease is unlikely to be an indication that the Pósa Method was not interesting for students.

Students agreed significantly less with the statement "I like creating my own math problems" in 2021 ($M = 2.02, SD = 1.27$) compared to 2018 ($M = 3.77, SD = 0.89), t(89) = 7.6, p < .001$. Since the Pósa Method seeks to teach students the value of a good question and encourages them to create their own follow up questions, this result is antithetical to the spirit of the Pósa Method. Two hypotheses for why this may be the case are the following. The first hypothesis is that students are so interested in the classroom problems that they do not feel a need to create their own problems. The second hypothesis is that students are not including their creation of problems as part of the class curriculum within their self-assessment of how much they like creating their own math problems. Similarly, students might not recognize their creation of math problems in the classroom, as according to teacher interviews, this process was phrased as "What would be a good next question?". That could lead them to only consider how much they like creating their own problems beyond what they already do in the class. If the second hypothesis is true, then perhaps there is little cause for concern. However, if the first hypothesis is true, then there may be a discrepancy between an objective of the Pósa Method and the outcome.

Students showed a significant increase in their agreement with the statement "I like being told how to solve a problem." between 2018 ($M = 2.06, SD = 1.17$) and 2021 ($M = 3.02, SD = 1.09$), $t(89) = -4.1, p < .001$. As the Pósa Method aims to give students enough time to think about the problems and discover ideas on their own, this increase appears to conflict with the spirit of the Pósa Method. However, given the aforementioned increase in agreement with the statement "I enjoy figuring out solutions by myself," a desire to learn about the solution before they have time to exhaust their own mathematical ideas and perseverance leads to a contradiction. A potential resolution to this contradiction is that student response to this question might be evaluating an experience more akin to learning about a satisfying solution. Without further research, this is difficult to clarify.

The repeated question: "What amount of time did you spend working on or thinking about problems posed in class out of class per week (e.g. writing homework or thinking about a problem)?" was not part of the Likert Scale, we can also obtain some insight on how much students think about mathematics problems. While the majority of students continue to spend half an hour to one hour thinking outside of class, there is a migration towards the right end of the distribution that provides evidence for greater engagement with thinking about problems (Appendix D).

Conclusion

The overarching goal of this project is to evaluate the efficacy of the Pósa Method in more typical math classrooms, rather than gifted programs. Overall, students in three secondary Hungarian classrooms had a positive experience learning mathematics with the Pósa Method. Based on student testimonies, there is evidence that the two main goals of the Pósa Method, enjoyment of mathematics and autonomous thinking, were achieved. In particular, the shift in relevant responses from 2018 to 2021 suggests that the Pósa Method successfully cultivates critical and independent thinking, as well as creative problem-solving skills. Despite its popularity, the Pósa Method was not favored by every student, which suggests that there may be ways to make it more effective for different types of learners. As with all novel teaching methods, further research is required to fine tune details and justify its widespread implementation. However, it is important to strike a balance between incorporating suggestions for improvement without compromising the intended structure of the Pósa Method.

It is important to note that while these three classrooms were not as selective as the Pósa camps, they are still regarded as "above average" schools in the capital of Hungary. As such, an interesting direction for future research would be to explore the implementation of the Pósa Method in more diverse contexts in terms of school type, academic level, location, and so on. Another possible direction for future research would be to investigate how the Pósa Method impacted the development of students' mathematical ability. This study focuses on students' perceptions, beliefs, and attitudes toward the Pósa Method, but it does not assess their problem solving skills or how those skills might have changed over time. An additional direction for future research is to examine students' results from the Matura mathematics final exam, which was administered shortly after the student survey. It would be interesting to analyze students' Matura exam scores and distribute a post-exam survey about how students thought the Pósa Method directly affected their exam preparation. Though this study suggests hopeful outcomes for students learning with the Pósa Method, there is more research to be done to comprehensively evaluate its efficacy. The MTA-Rényi Research Group is conducting a longitudinal study of learning outcomes and experiences of students learning with the Pósa Method that will provide more insight into the adequacy of the Pósa Method in different contexts.

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Appendix A

Response	Number of students (%)
1-horrible	1 (2.3)
2-bad	1 (2.3)
3-ok	2 (4.5)
4-good	12 (27.2)
5-great	28 (63.6)

Response	Number of students (%)
1-very easy	0 (0)
2-moderately easy	13 (29.5)
3-average	23 (52.3)
4-moderately difficult	8 (18.2)
5-very difficult	0 (0)

Table 1. Responses to "Based on my experiences, discovery learning was. . . "

Appendix B

Question & Response	2018 mean	2021 mean	2018 standard deviation	2021 standard deviation	t test
How interesting (in general) were your math classes? *(1-very uninteresting, 5-very interesting)*	4.21	4.02	0.72	0.9	$p = 0.27$
To what extent did your grades reflect your effort in the class? *(1-My effort should have earned a much higher grade, 5-My lack of effort should have earned a much lower grade)*	3.91	3.36	0.65	0.65	$p = 0.21$
To what extent did your grades reflect your math knowledge? *(1-My math knowledge should have earned a much higher grade, 5-My math knowledge should have earned a much lower grade)*	2.98	3.07	0.44	0.62	$p = 0.44$
I understand the math I learned from 9th to 11th grade. *(1-strongly disagree, 5-strongly agree)*	4.15	4.32	0.69	0.67	$p = 0.24$
I like coming up with and solving my own math questions. *(1-strongly disagree, 5-strongly agree)*	2.62	2.8	0.99	1.21	$p = 0.45$
I am good at solving difficult math problems. *(1-strongly disagree, 5-strongly agree)*	3.47	3.27	1.02	0.92	$p = 0.34$

Table 2. Student survey responses with no statistically significant change

Appendix C

Question & Response *(5-point Likert scale: 1-strongly disagree, 5-strongly agree)*	2018 mean	2021 mean	2018 standard deviation	2021 standard deviation	t test
In life, a problem can be approached from different angles.	4.43	4.8	0.89	0.59	$p = 0.0203$
If I face a problem, I try to understand it from different angles.	3.3	4.5	0.88	0.82	$p = 1.59x10^{-9}$
It is possible to arrive at the correct solution in multiple ways.	2.6	4.77	1.26	0.6	$p = 7.87x10^{-16}$
I can arrive at the correct solution of a math problem in different ways.	3.32	3.91	0.98	0.91	$p = 0.0038$
I enjoy figuring out solutions by myself.	3.28	4.43	0.97	0.85	$p = 3.26x10^{-8}$
I like creating my own math problems.	3.77	2.02	0.89	1.27	$p = 7.62x10^{-11}$
I like being told how to solve a problem.	2.06	3.02	1.17	1.09	$p = 0.0001$
I like practicing the same procedure through multiple math problems.	4.51	3.86	0.75	1.03	$p = 0.0009$
How interesting were the math problems? *(1-very uninteresting, 5-very interesting)*	4.3	3.89	0.83	0.72	$p = 0.013$

Table 3. Student survey responses with a statistically significant change

Appendix D

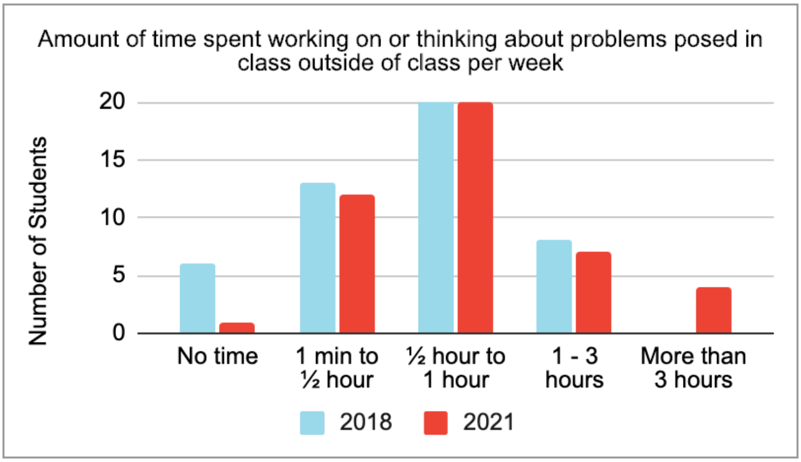

Figure 1. Student survey responses

References

Artigue, M., Bosch, M., Doorman, M., Juhász, P., Kvasz, L., & Maass, K. (2020). Inquiry based mathematics education and the development of learning trajectories. *Teaching Mathematics and Computer Science*, *18*(3), 63–89. doi: 10.5485/tmcs.2020.0505

Creswell, J. (2009). *Research design: qualitative, quantitative, and mixed methods approaches*. SAGE Publications.

Gosztonyi, K. (2015). *Tradition and reform in mathematics education during the "New Math" period: A comparative study of the case of Hungary and France*. [Unpublished doctoral dissertation]. University of Szeged.

Gosztonyi, K. (2016). Mathematical culture and mathematics education in Hungary in the XXth century. In B. Larvor (Ed.), *Mathematical cultures. Trends in the history of science* (pp. 71–89). Birkhäuser. doi: 10.1007/978-3-319-28582-5_5

Gosztonyi, K. (2020). Tamás Varga's reform movement and the Hungarian Guided Discovery approach. *Teaching Mathematics and Computer Science*, *18*(3), 11–28. doi: 10.5485/tmcs.2020.0475

Győri, J. G., & Juhász, P. (2017). An extra-curricular gifted support programme in Hungary for exceptional students in mathematics. In K. S. Taber,

M. Sumida, & L. McClure (Eds.), *Teaching gifted learners in STEM subjects* (pp. 89–106). Rutledge. doi: 10.4324/9781315697147-7

Honomichl, R. D., & Chen, Z. (2012). The role of guidance in childrens' discovery learning. *WIREs Cognitive Science*, *3*(6), 615–622. doi: 10.1002/wcs.1199

Katona, D., & Szűcs, G. (2017). Pósa Method & cubic geometry – A sample of a problem thread for discovery learning of mathematics. In T. J. Karlovitz (Ed.), *Differences in pedagogical theory and practice* (pp. 17–34). International Research Institute sro, Komárno. doi: 10.18427/iri-2017-0079

Kile, S., Kokkila, L., & Long, P. (n.d.). *Students' perceptions of the discovery learning. Pósa Method in three secondary math groups in Hungary.* [Unpublished]. Budapest Semesters in Mathematics Education.

Matzal, C., Manahan, K., Galaty, B., Wang, H., & Barbarics, M. (2020). Guided discovery in Hungarian education using problem threads: The Pósa Method in secondary mathematics classrooms. *Teaching Mathematics and Computer Science*, *18*(1), 51–67. doi: 10.5485/tmcs.2020.0491

Stockton, J. C. (2010). Education of mathematically talented students in Hungary. *Journal of Mathematics Education at Teachers College*, *1*(2). doi: 10.7916/jmetc.v1i2.682

Strauss, A., & Corbin, J. (1991). *Basics of qualitative research: grounded theory procedures and techniques.* SAGE Publications.

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