



On some problems on composition of arithmetic functions

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Abstract. The main goal of this paper is to investigate some problems related to the commutativity of the composition of arithmetic functions. The concept of commutativity arises many times in high school maths, so it is natural to study the composition of functions, namely the equation $f(g(n)) = g(f(n))$, where f and g are such well known arithmetic functions as $d(n)$, $\varphi(n)$, $\sigma(n)$, $\omega(n)$, or $\Omega(n)$. We study various aspects of solvability: can we exhibit infinitely many solutions; can we determine every solution; can we find suitable values in the range of both functions f and g for which the equation is, or is not solvable, respectively. We need just the basic facts about the above functions, and we use only elementary methods in the proofs. We present some interesting questions, their solutions, and raise some unsolved problems. We found that this topic can be discussed well in secondary school, mainly within the framework of group study sessions as we had some classes with a group of kids in 9th grade. We summarize the experiences of this experiment in the last section.

Key words and phrases: arithmetic functions, composition, commutativity.

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